

QED Based on Self-Fields: A Relativistic Calculation of $g-2$.

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A recently advanced theory of quantum electrodynamics which is not second quantized, but rather based on self-fields is adopted to a relativistic calculation of ($g-2$). In analogy to classical electrodynamics, radiative corrections are seen as arising from the back reaction of the self-field upon the source. Vacuum field fluctuations, assumed to be the *physical cause* of radiative correction in standard QED, are absent in the present self-field approach, which recently has been applied to calculate spontaneous emission, Lamb shift, vacuum polarization, and to a non-relativistic calculation of ($g-2$), all in free space as well as in cavities. We conclude that the self-field of the electron can be consistently considered to be the physical origin of all radiative processes, as an alternative picture to hypothetical vacuum fluctuations.

Key words: Quantum electrodynamics, Self-fields, $g-2$, Second quantization.

I. Introduction

In quantum electrodynamics (QED) the second quantization of the electromagnetic (EM) field necessarily implies the existence of zeropoint fluctuations of electric and magnetic field strengths in the vacuum. These fluctuations are interpreted as being the physical origin of such radiative effects as Casimir effect, spontaneous emission, Lamb shift and the nonzero value of $g-2$ in freespace. The scale of these zeropoint fluctuations is set by the constant $\hbar c$, and consequently they vanish in the limit $\hbar \rightarrow 0$, yielding the truly empty classical vacuum. In this scenario one would expect that such phenomena as spontaneous emission and Lamb shift would also disappear in this limit $\hbar \rightarrow 0$, since their *physical cause* – the vacuum field – is no longer present. However this is not the case. Spontaneous emission and the Lamb shift have perfectly respectable classical analogues in the occurrence of line broadening and level shifts which are predicted by the classical theory of radiation reaction [1]. Hence it appears that the standard interpretation of radiative corrections on QED does not meet with the spirit of the *correspondence principle*.

In the classical theory, line broadening and level shifts occur in a system such as a harmonically bound charge when the back reaction of the self-field of the charge is included in the equations of motion – as is required by the self-consistency of the theory. One

might expect that it should be possible to construct a complete theory of QED in direct analogy to classical radiation reaction theory; whereby one adds to the Dirac equation of motion a term which includes in a self-consistent manner the self-field generated by the electron's fourcurrent itself. In such a theory the back reaction of the self-field upon the fourcurrent would be viewed as the physical origin of spontaneous emission, the Lamb shift, and $g-2$, and other radiative processes with a direct classical correspondence to similar effects.

Early in the history of QED precisely such a self-field approach was attempted by Schrödinger [2] and also by Fermi [3]. Schrödinger insisted that the self-consistency of the theory required the addition of a self-field term to the quantum mechanical (QM) equation of motion – just as such a term is required for the consistency of the Abraham-Lorentz-Dirac equation of motion for a point charge. In the Schrödinger interpretation of QM, revived recently [4], the field $e\Psi$ is a measure for the actual physical distribution of the electron charge and not just a probability amplitude; hence the inclusion of the self-field is seen as a necessary requirement. By including a classically modelled radiation reaction term in the Schrödinger equation, Fermi was able to derive the Einstein A coefficient for spontaneous emission. Fermi's calculation was carried out in 1927; the same year as when Dirac derived the A coefficient by second quantizing the EM field [5]. The self-field approach to QED was taken up by Jaynes and his collaborators [6] as the neoclassical theory of radiation. The theory in its original form

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proved to be at most a semiclassical approximation to a full theory of QED, but nevertheless Jaynes was able to show that one could go quite far in accounting for radiative effects without having to resort to the second quantizing of the radiation field.

More recently Barut and his collaborators [7] have proposed a complete self-consistent self-field approach to QED in which the EM self-field potential A produced by the fourcurrent ej^μ is added to the minimally coupled equation of motion for a point charge. Through the use of an EM Green's function one may eliminate totally A from the equation of motion leaving a nonlinear, nonlocal interaction term which when properly analyzed yields radiative corrections to the original energy spectrum and other radiative processes.

In the approach of Barut et al. neither the matter nor the EM fields are second quantized. In the fully relativistic version of the theory, Barut and Salamin [8] have given very precise formulae for the relativistic spontaneous emission rates and the Lamb shift in freespace. In the nonrelativistic (NR) approximation of the theory Barut and van Huele [9] have given derivations of the Einstein A coefficient and the NR Bethe formula for the Lamb shift. Barut, Dowling, and van Huele [10] have given a NR derivation of the freespace value of $g-2$ which is in many ways superior to similar NR calculations made in standard QED [11]. In a series of papers Barut and Dowling have shown how the self-field approach can satisfactorily account for apparatus dependent contributions to spontaneous emission rates, to the Lamb shift, Casimir and Casimir-Polder effects, and to $g-2$ [12].

The electron's anomaly ($g-2$) was first calculated by Schwinger [13] in quantum electrodynamics in the form of an additional mass matrix in the Dirac equation resulting from the quantized radiation field; it is equivalent to a lowest order Feynman graph calculation. A different perturbative calculation was given in S -matrix formulation [14]. Babiker [15] calculated ($g-2$) in the semi-classical radiation theory by treating the field classically but the electron in a second quantized theory. Later Schwinger gave another calculation of ($g-2$) in his source theory [16]. Our calculation turns out to be very close to Babiker's and we shall make use of some of his integrals.

In the present work we give a fully relativistic derivation of the first order correction to $g-2$ in freespace in the self-field approach to QED by Barut and his coworkers thereby developing this formalism to-

wards a more complete and self-consistent theory. Here we obtain $g-2$ in a theory where both the matter and EM fields are c -numbers. We further indicate higher order terms for ($g-2$) in the present approach.

II. The Self-Field Approach to QED

In the self-field approach to quantum electrodynamics (which we shall abbreviate self field electrodynamics (SED)) the fundamental starting point of view is that electromagnetic fields do not exist independently of the currents which produce them. In a region of space outside the light cone of any relevant electromagnetic sources the vacuum field strengths of the electric and magnetic fields are assumed to be identically zero – just as in classical electrodynamics (CED). In SED the vacuum does not fluctuate.

It is assumed in practical calculations that the total electromagnetic field potential A_μ surrounding a charge can be conceptually separated into superposition of a self-field A_μ^s originating in the charge itself, and an external field A_μ^e originating from sources far removed from the original charge. For instance, A_μ^e might be the Coulomb potential of an infinitely massive nucleus, or the potential of an externally imposed homogeneous magnetic field.

We define as usual the electromagnetic field tensor as

$$F_{\mu\nu} := A_{[\nu, \mu]}, \quad (1)$$

where $[\nu, \mu]$ indicates antisymmetrization with respect to the indices ν and μ . Due to the linearity of (1) the total $F_{\mu\nu}$ may also be written as the sum of a self-field and an external field tensor. The self-field tensor obeys Maxwell's inhomogeneous equation

$$F_{\nu, \mu}^{\mu\nu} = ej^\nu, \quad (2)$$

where

$$ej^\nu := e\bar{\Psi}\gamma^\nu\Psi \quad (3)$$

is the usual Dirac fourcurrent ($e > 0$). In order to work in a theory which is manifestly covariant we choose the Lorentz gauge condition

$$A_{\mu,}{}^\mu \equiv 0. \quad (4)$$

Once the choice of gauge is made (2) may be solved for A_μ^s with the aid of an electromagnetic Green's function $D(x-y)$

$$A_\mu^s(x) = e \int dy D(x-y) j_\mu(y), \quad (5)$$

where $dy := d^4y$, $x := x^\mu$, etc. If one makes use of the conservation of current equation $k_\mu j^\mu \equiv 0$ one can easily show that the gauge condition (4) is automatically satisfied if one chooses the Green's function

$$D(x-y) = \frac{1}{(2\pi)^4} \int dk \frac{e^{-ik \cdot (x-y)}}{k^2 + i\varepsilon}, \quad (6)$$

where the $i\varepsilon$ insures that we have chosen $D(x-y)$ to correspond to the usual Stueckelberg-Feynman Green's function appropriate for the Dirac equation with antiparticles.

III. Action Formalism

In the theory of SED as proposed by Barut and his coworkers, radiative corrections to energy levels of isolated quantum systems are most easily computed directly from the action integral without solving the equations of motion. Let a lower case w represent an action density and an uppercase W the total action. One then defines

$$W := \int dx w[x, A(x), \Psi(x)], \quad (7)$$

where $dx = d^4x$, $x = x^\mu$, etc. For a Dirac particle we separate the action density into the sum of a free matter field density w_{fp} (free particle), an interaction term w_i and a free EM field contribution w_f ; $w = w_{\text{pf}} + w_i + w_f$, which are given as

$$\begin{aligned} w_{\text{pf}} &:= \bar{\Psi} \{i\gamma^\mu \partial_\mu - m\} \Psi = \bar{\Psi} \{\gamma^\mu P_\mu - m\} \Psi, \\ w_i &:= \bar{\Psi} \{e\gamma^\mu A_\mu\} \Psi = e j^\mu A_\mu, \\ w_f &:= \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \end{aligned} \quad (8)$$

where we recall that $A_\mu = A_\mu^e + A_\mu^s$ and $F_{\mu\nu} = F_{\mu\nu}^e + F_{\mu\nu}^s$.

Variation of the total action yields the Euler-Lagrange equations. Variation with respect to the Dirac field yields

$$\frac{\delta W}{\delta \bar{\Psi}} - \partial_\mu \frac{\delta W}{\delta \bar{\Psi}_{,\mu}} = \{\gamma^\mu P_\mu + e\gamma^\mu A_\mu\} \Psi = 0, \quad (9)$$

the Dirac equation of motion. Variation with respect to the EM self-field gives

$$\frac{\delta W}{\delta A_\nu^s} - \partial_\mu \frac{\delta W}{\delta A_{\nu,\mu}^s} = -e j^\nu + F_s^{\mu\nu}{}_{,\mu} = 0, \quad (10)$$

which is just the inhomogeneous Maxwell equation (2) with the current j^ν defined as in (3).

For a scattering problem let us call the scattering amplitude per unit spacetime \mathcal{G} – and for a bound-state problem the total invariant energy of the system \mathcal{E} . The action W can then be related to these two directly measurable quantities via

$$W_{\text{fi}} = (2\pi)^4 \delta^4(P_f - P_i) \mathcal{G}, \quad (11a)$$

$$W_{\text{fi}} = (2\pi) \delta(E_f - E_i) \mathcal{E}, \quad (11b)$$

where f and i stand for the final and initial values of the fourmomentum P or the total energy level E .

By linearity, the EM field action density w_f of (8) can be written as

$$w_f = \frac{1}{4} \{F_{\mu\nu}^s F_s^{\mu\nu} + F_{\mu\nu}^e F_e^{\mu\nu} + F_{\mu\nu}^e F_s^{\mu\nu} + F_{\mu\nu}^s F_e^{\mu\nu}\}. \quad (12)$$

The two middle terms may be converted to surface integrals which vanish for bound state problems, i.e. if it is presupposed that $A_\mu^s(x)$ vanishes sufficiently fast at infinity in Minkowski space. One keeps the surface terms for the radiation going to infinity. (Throughout this paper action densities $w(x)$ will be considered equal so long as they are equal modulo integration by parts and possible surface terms when they vanish as $|x|$ and $|x_0|$ tend to infinity.) The last term in (11) is the usual field invariant

$$\frac{1}{4} F_{\mu\nu}^e F_e^{\mu\nu} = -\frac{1}{2} (E_e^2 - B_e^2), \quad (13)$$

which we shall henceforth drop from the total action as it is a nondynamical constant. Only the first term of (11) remains, and it may be transformed via integration by parts as

$$\begin{aligned} \frac{1}{4} F_{\mu\nu}^s F_s^{\mu\nu} &= \frac{1}{4} A_{[\mu,\nu]}^s F_s^{\mu\nu} \\ &= \frac{1}{4} [A_{[\nu}^s F_{\mu]}^{\mu\nu}]_{,\mu} - \frac{1}{4} A_{[\nu}^s F^{\mu\nu}{}_{s,\mu]} \\ &= -\frac{e}{2} A_\nu^s j^\nu, \end{aligned} \quad (14)$$

where we have dropped a surface term and used the inhomogeneous Maxwell equation (2). Equation (14) allows us to combine w_i and w_f of (8) to get for the total action density

$$\begin{aligned} w &:= w_0 + w', \\ w_0 &:= \bar{\Psi} \{\gamma^\mu P_\mu - m\} \Psi + e j^\mu A_\mu^e, \\ w' &:= \frac{e}{2} j^\mu A_\mu^s. \end{aligned} \quad (15)$$

Here w_0 will give rise to the action and equations of motion of a Dirac electron interacting with an external potential A_μ^e , and w' will be responsible for introducing radiative corrections arising from the interaction of the electron with its own field.

IV. Elimination of the Self-Field

We may now use (5), which gives A_μ^s in terms of j_μ , to eliminate A_μ^s altogether from the total action. Defining $W' = \int dx w'(x)$, where $w'(x)$ is given in (15), we have

$$\begin{aligned} W' &= \int dx w'(x) \\ &= \int dx \left\{ \frac{e}{2} j^\mu(x) A_\mu^s(x) \right\} \\ &= \frac{e^2}{2} \iint dx dy j^\mu(x) D(x-y) j_\mu(y), \end{aligned} \quad (16)$$

where $D(x-y)$ is given in (6). The self-field contribution to the total action W' given in (16) is nonlinear and nonlocal; in SED all quantum electrodynamical effects arise from this piece of the action.

To proceed with the analysis of W' we perform a Fourier expansion of the Dirac field,

$$\Psi(x) = \int dp e^{-ip \cdot x} \Psi(p). \quad (17)$$

This expansion has to be understood in a general sense to include, in general, the discrete spectrum in the external field: $\int dp_0$ may contain a sum as well as an integration.

Let us now expand the Ψ_s' which appear in (16) to obtain

$$\begin{aligned} W' &= \frac{e^2}{2} \frac{1}{(2\pi)^4} \iiint dx dy dk dp dq dr ds \\ &\quad \times \bar{\Psi}(p) \gamma^\mu \Psi(q) \frac{e^{-ik \cdot (x-y)}}{k^2 + i\epsilon} \bar{\Psi}(r) \gamma_\mu \Psi(s) \\ &\quad \times \exp i \{ (p-q) \cdot x + (r-s) \cdot y \}. \end{aligned} \quad (18)$$

If we now carry out the dx, dy and dk integrations, the exponential factor gives rise to a delta function, viz.

$$\begin{aligned} W' &= \frac{e^2}{2} (2\pi)^4 \iiint dp dq dr ds \bar{\Psi}(p) \gamma^\mu \Psi(q) \\ &\quad \times \frac{1}{(r-s)^2 + i\epsilon} \Psi(r) \gamma_\mu \Psi(s) \delta^4(p-q+r-s). \end{aligned} \quad (19)$$

The delta function is satisfied by the two distinct conservation of energy-momentum conditions

$$p = q, \quad r = s, \quad (I)$$

$$p = s, \quad q = r. \quad (II)$$

Condition (I) and the corresponding action W_I' lead to an effect similar to "vacuum polarization" in standard

QED. Here we shall concentrate on condition (II), which leads to the self-energy correction responsible for g -2.

Now that we have shown that the term (II) is required, we may go back and separate it retroactively to expression (18) before carrying out the dx, dy and dk integrations. Hence (18) with the additional condition (II) can be written

$$\begin{aligned} W_{II}' &= \frac{e^2}{2} \int dp \iint dx dy D(x-y) \bar{\Psi}(p) e^{ip \cdot x} \\ &\quad \times \gamma^\mu \left\{ \int dq \Psi(q) \bar{\Psi}(q) e^{i(y-x) \cdot q} \right\} \gamma_\mu \Psi(p) e^{-ip \cdot y}. \end{aligned} \quad (20)$$

The integral $\int dq$ which appears in the curly brackets of (20) is over both positive and negative energy states. However we must impose the additional boundary condition that positive energy solutions evolve forward in time and negative energy solutions backward. To insure this we demand the condition

$$\begin{aligned} \int dq \Psi(q) \bar{\Psi}(q) e^{i(y-x) \cdot q} \\ &= \Theta(y_0 - x_0) \int_{(+)} dq \Psi(q) \bar{\Psi}(q) e^{i(y-x) \cdot q} \\ &\quad - \Theta(x_0 - y_0) \int_{(-)} dq \Psi(q) \bar{\Psi}(q) e^{i(y-x) \cdot q} \\ &=: S(x-y), \end{aligned} \quad (21)$$

where $\int_{(\pm)} dq$ stands for an integral over positive or negative energy states, and Θ is the usual step function. If the Ψ are taken as exact solutions which minimize the action $W_0 = \int dx w_0(x)$, with $w_0(x)$ given in (15), then $-iS(x-y)$ is the usual Feynman Green's function for the Dirac equation of an electron in whatever external EM field we have:

$$[\gamma^\mu p_\mu - e \gamma^\mu A_\mu - m] S(x-y) = i \delta(x-y). \quad (22)$$

We should remark that there is a consistent treatment of antiparticles in the first quantized Dirac theory. One can use two Dirac equations, $(\gamma \cdot p - m) \Psi = 0$ and its mass conjugate, $(\gamma \cdot p + m) \bar{\Psi} = 0$. The negative energy solutions of the first equation correspond to the positive energy solutions of the second equation. In the presence of the minimal coupling the second equation couples with $(-e)$ and $(-p_\mu)$ relative to the first, leading to the Stueckelberg-Feynman rule of propagation backwards in time of negative energy states used in (21).

In momentum space we may define implicitly $S(p)$, as usual, as

$$S(x-y) =: \frac{1}{(2\pi)^4} \int dp e^{-ip \cdot (x-y)} S(p). \quad (23)$$

The operator $S(q)$ may be obtained by inverting the Dirac operator $\gamma^\mu \pi_\mu - m$ where $\gamma^\mu \pi_\mu := \gamma^\mu p_\mu - e \gamma^\mu A_\mu$. This is mostly easily done through the use of the Heaviside operator calculus. To order e^2 one obtains

$$\begin{aligned} \frac{1}{\gamma^\mu \pi_\mu - m} &= \frac{\gamma^\mu P_\mu - e \gamma^\mu A_\mu + m}{P^2 - m^2} + 2e \frac{P \cdot A (\gamma^\mu P_\mu + m)}{(P^2 - m^2)^2} \\ &\quad - ie \frac{(\gamma^\mu P_\mu + m) \gamma^\mu \gamma^\nu F_{\mu\nu}}{(P^2 - m^2)^2} - 2e^2 \frac{\gamma^\mu A_\mu (P \cdot A)}{(P^2 - m^2)^2} \\ &\quad + e^2 A^2 \frac{\gamma^\mu P_\mu + m}{(P^2 - m^2)^2} + ie^2 \frac{\gamma^\mu A_\mu \gamma^\nu F_{\mu\nu}}{(P^2 - m^2)^2} \\ &\quad + 4e^2 \frac{(\gamma^\mu P_\mu + m)(P \cdot A)^2}{(P^2 - m^2)^3} + O(e^3). \end{aligned} \quad (24)$$

Keeping this expansion in mind, we write (20) as

$$W_{II}' = -\frac{ie^2}{2} \iint dp dP \bar{\Psi}(p) \frac{\gamma^\mu S(p) \gamma_\mu}{(p-P)^2 + i\varepsilon} \Psi(p). \quad (25)$$

By the prescription of (11a) we may convert this contribution to the total action W' into an energy shift via

$$\begin{aligned} \delta E &= \frac{W_{II}'}{(2\pi)^4} \\ &= -\frac{ie^2}{2} \frac{1}{(2\pi)^4} \iint dp dP \bar{\Psi}(p) \frac{\gamma^\mu S(p) \gamma_\mu}{(p-P)^2 + i\varepsilon} \Psi(p) \\ &= \int dp \bar{\Psi}(p) \delta M(p) \Psi(p), \end{aligned} \quad (26)$$

where

$$\begin{aligned} \delta M(p) &:= -\frac{e^2}{2} \frac{i}{(2\pi)^4} \int dP \frac{\gamma^\mu S(P) \gamma_\mu}{(p-P)^2 + i\varepsilon} \quad \left(\begin{array}{l} s = p - P \\ ds = -dP \end{array} \right) \\ &= \frac{e^2}{2} \frac{i}{(2\pi)^4} \int ds \frac{\gamma^\mu S(p-s) \gamma_\mu}{s^2 + i\varepsilon}, \end{aligned} \quad (27)$$

which is identical to the operator ΔM obtained by Babiker [15].

V. Evaluation of $(g-2)$ and Energy Shifts

It remains now to evaluate the mass-shift operator $\delta M(p)$ in (27) and with it δE in (26). In the absence of a closed formula for the propagator $S(p)$ in an external field we shall use the power series expansion of it in e given in (24). As Babiker has shown, the insertion of (24) into (27) leads to standard four-dimensional Feynman integrals. Aside from a δ -function potential

(which Babiker does not include), which is not relevant for the $(g-2)$ (but is of course very important for the S -wave Lamb shift), the lowest non-vanishing contribution comes from the third term in (24), namely

$$-ie \frac{(\gamma^\mu P_\mu + m) \gamma^\mu \gamma^\nu F_{\mu\nu}}{(P^2 - m^2)^2}.$$

We note that this term is gauge-invariant. All other gauge-noninvariant terms involving A_μ give automatically vanishing contributions. The free particle propagator which corresponds to renormalization terms (or to regularization procedures) is subtracted. The result of (27) to lowest order in e in the propagator is thus

$$\begin{aligned} \delta M(p) &= \frac{e^2}{2} \frac{i}{(2\pi)^4} (-ie) \frac{2}{m} \int_0^1 dy (1-y) \sigma^{\mu\nu} F_{\mu\nu} \\ &= -\frac{\alpha}{2\pi} \frac{e}{2m} \sigma^{\mu\nu} F_{\mu\nu} \\ &= -\frac{\alpha}{2\pi} \frac{e}{2m} (\boldsymbol{\sigma} \cdot \mathbf{B} + i \boldsymbol{\alpha} \cdot \mathbf{E}), \quad \left(\alpha = \frac{e^2}{4\pi} \right). \end{aligned} \quad (28)$$

We thus read off $(g-2)$ to lowest order in the Green's function as $\alpha/2\pi$. The matrix element of (28), when inserted into (26), gives the Lamb shift δE . It is well-known that, again to this lowest order, the Lamb shift is given by this matrix element, apart from the δ -function potential mentioned above.

IV. Discussion and Conclusions

Equation (26) is in fact a general formula for the Lamb shift in any external field. Thus, in principle, $(g-2)$ can be read off in any external field problem, not necessarily an external magnetic field. For example, in the Coulomb-field of the H-atom there is a magnetic part of the Lamb shift due to a term in δM proportional to $\boldsymbol{\sigma} \cdot \mathbf{B}$ whose coefficient gives us the $(g-2)$ factor immediately even before evaluating the energy shift δE itself.

The full evaluation of δE according to (26) requires the knowledge of the Green's function $S(P)$ in the external field, which is in general difficult and unknown. The direct calculation of δE by summing over $\int dq$ in (20) is in fact equivalent to a determination of the Green's function. For the relativistic Coulomb problem this problem has been considerably advanced up to some final numerical integrations [17].

However, in an iterative calculation, the expansion of $S(P)$ in powers of E yields, as we have seen, in a relatively simple way a calculation of $(g-2)$ to order α . The next order terms will come from the higher order terms in the propagator as well as from the change $\delta\Psi$

of the (Coulomb) wave functions due to self-energy. We have a well defined problem so that a nonperturbative numerical calculation of $(g-2)$ is in principle possible, which seems to be far simpler than the thousands of graphs of standard perturbation theory.

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